

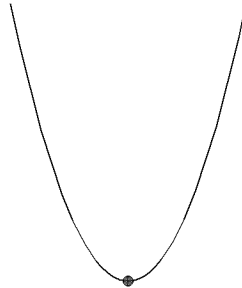
[07-10-31-T11]

*Translation of figures in the plane*

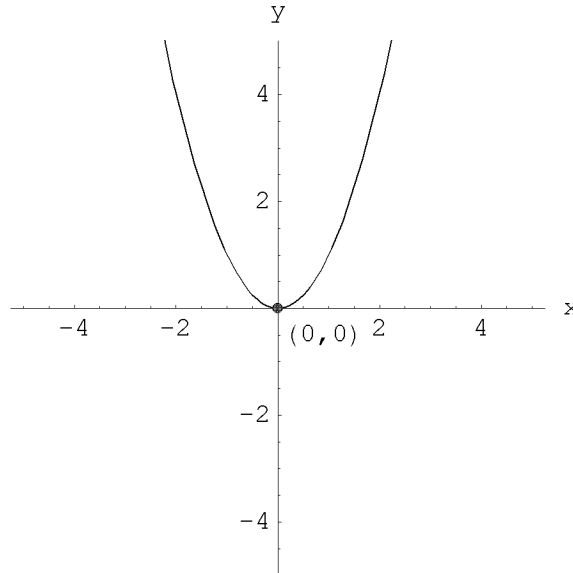
---

■ **It's the coordinate system that moves, not the figure!**

Consider the following parabola. Its vertex is identified by the large dot.

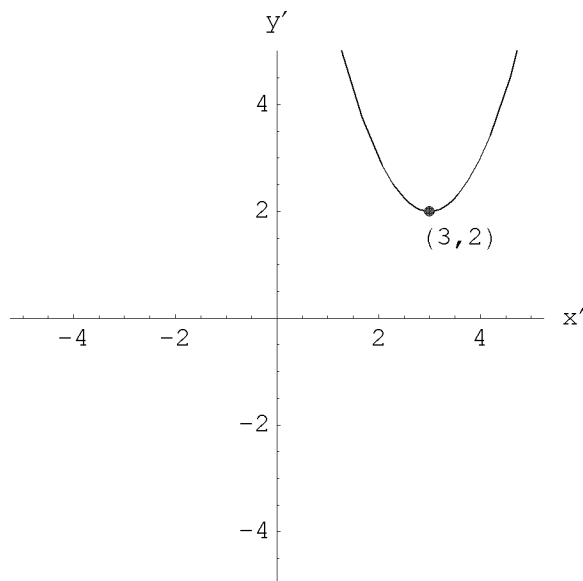


Now overlay a coordinate system. You are free to place the coordinate system where you wish.



In this example, we happen to have placed the origin of the  $xy$  system at the vertex of the parabola. In the  $xy$  system, the vertex has coordinates  $(0, 0)$ . The function  $y = x^2$  would produce the set of points  $(x, y)$  that make up this parabola.

Translate the  $xy$  coordinate system 3 units left and 2 units down. Call the shifted coordinate system the  $x'y'$  system.



In the  $x'y'$  system, the vertex has coordinates  $(3, 2)$ . The function  $y' - 2 = (x' - 3)^2$  would produce the set of points  $(x', y')$  that make up this parabola.

In general, if  $y = f(x)$  in a certain coordinate system, then the function is written  $y' - k = f(x' - h)$ , where  $h$  and  $k$  are positive numbers, when the coordinate system is translated  $h$  units left and  $k$  units down.

Once this is understood, we typically avoid the correct but cumbersome language " $y - 5 = (x - 3)^2$  is the function  $y = x^2$  represented in a coordinate system that has been translated 3 units left and 5 units down from the system in which the vertex lies at  $(0, 0)$ ." Instead, we say " $y - 5 = (x - 3)^2$  is the function  $y = x^2$  translated 3 units right and 5 units up."

#### ■ Example 1

The graph of  $y - 5 = (x - 1)^3$  is the graph of  $y = x^3$  translated 1 unit right and 5 units up.

#### ■ Example 2

The graph of  $(x - 7)^2 + (y - 2)^2 = 4$  is the graph of a circle radius 2 centered at  $(7, 2)$ .

#### ■ Example 3

The graph of  $y = (x - 9)^2 + 2$  is the graph of  $y = x^2$  shifted 9 units right and 2 units up. [Note:  $y = (x - 9)^2 + 2 \iff y - 2 = (x - 9)^2$ ].

#### ■ Example 4

The graph of  $y + 5 = (x + 2)^2$  is the graph of  $y = x^2$  shifted 2 units left and 5 units down. [Note:  $y + 5 = (x + 2)^2 \iff y - (-2) = (x - (-9))^2$ ].